

nag_regsn_mult_linear_tran_model (g02dkc)

1. Purpose

nag_regsn_mult_linear_tran_model (g02dkc) calculates the estimates of the parameters of a general linear regression model for given constraints from the singular value decomposition results.

2. Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_regsn_mult_linear_tran_model(Integer ip, Integer iconst, double p[],
    double c[], Integer tdc, double b[], double rss, double df, double se[],
    double cov[], NagError *fail)
```

3. Description

This function computes the estimates given a set of linear constraints for a general linear regression model which is not of full rank. It is intended for use after a call to **nag_regsn_mult_linear (g02dac)** or **nag_regsn_mult_linear_upd_model (g02ddc)**.

In the case of a model not of full rank the functions use a singular value decomposition (SVD) to find the parameter estimates, $\hat{\beta}_{svd}$, and their variance-covariance matrix. Details of the SVD are made available, in the form of the matrix P^* :

$$P^* = \begin{pmatrix} D^{-1}P_1^T \\ P_0^T \end{pmatrix}$$

as described by **nag_regsn_mult_linear (g02dac)** and **nag_regsn_mult_linear_upd_model (g02ddc)**.

Alternative solutions can be formed by imposing constraints on the parameters. If there are p parameters and the rank of the model is k , then $n_c = p - k$ constraints will have to be imposed to obtain a unique solution.

Let C be a p by n_c matrix of constraints, such that

$$C^T \beta = 0,$$

then the new parameter estimates $\hat{\beta}_c$ are given by:

$$\begin{aligned} \hat{\beta}_c &= A\hat{\beta}_{svd} \\ &= (I - P_0(C^T P_0)^{-1})\hat{\beta}_{svd}, \end{aligned}$$

where I is the identity matrix, and the variance-covariance matrix is given by:

$$AP_1 D^{-2} P_1^T A^T$$

provided $(C^T P_0)^{-1}$ exists.

4. Parameters

ip

Input: the number of terms in the linear model, p .

Constraint: **ip** ≥ 1 .

iconst

Input: the number of constraints to be imposed on the parameters, n_c .

Constraint: **0** < **iconst** < **ip**.

p[ip*ip+2*ip]

Input: **p** as returned by **nag_regsn_mult_linear (g02dac)** and **nag_regsn_mult_linear_upd_model (g02ddc)**.

c[ip][tdc]

Input: the **iconst** constraints stored by column, i.e., the i th constraint is stored in the i th column of **c**.

tdc

Input: the last dimension of the array **c** as declared in the function from which nag_regsn_mult_linear_tran_model is called.

Constraint: **tdc** \geq **iconst**.

b[ip]

Input: the parameter estimates computed by using the singular value decomposition, $\hat{\beta}_{svd}$.
Output: the parameter estimates of the parameters with the constraints imposed, $\hat{\beta}_c$.

rss

Input: the residual sum of squares as returned by nag_regsn_mult_linear (g02dac) or nag_regsn_mult_linear_upd_model (g02ddc).

Constraint: **rss** $>$ 0.0

df

Input: the degrees of freedom associated with the residual sum of squares as returned by nag_regsn_mult_linear (g02dac) or nag_regsn_mult_linear_upd_model (g02ddc).

Constraint: **df** $>$ 0.0.

se[ip]

Output: the standard error of the parameter estimates in **b**.

cov[(ip*(ip+1)/2]

Output: the upper triangular part of the variance-covariance matrix of the **ip** parameter estimates given in **b**. They are stored packed by column, i.e., the covariance between the parameter estimate given in **b**[i] and the parameter estimate given in **b**[j], $j \geq i$, is stored in **cov**[$j(j+1)/2 + i$], for $i = 0, 1, \dots, \mathbf{ip} - 1$ and $j = i, i + 1, \dots, \mathbf{ip} - 1$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_INT_ARG_LT

On entry, **ip** must not be less than 1: **ip** = $\langle value \rangle$.

NE_INT_ARG_LE

On entry, **iconst** must not be less than or equal to 0: **iconst** = $\langle value \rangle$.

NE_2_INT_ARG_GE

On entry, **iconst** = $\langle value \rangle$ while **ip** = $\langle value \rangle$. These parameters must satisfy **iconst** $<$ **ip**.

NE_2_INT_ARG_LT

On entry, **tdc** = $\langle value \rangle$ while **iconst** = $\langle value \rangle$. These parameters must satisfy **tdc** \geq **iconst**.

NE_REAL_ARG_LE

On entry, **rss** must not be less than or equal to 0.0: **rss** = $\langle value \rangle$.

On entry, **df** must not be less than or equal to 0.0: **df** = $\langle value \rangle$.

NE_MAT_NOT_FULL_RANK

Matrix **c** does not give a model of full rank.

NE_ALLOC_FAIL

Memory allocation failed.

6. Further Comments

This function is intended for use in situations in which dummy (0-1) variables have been used such as in the analysis of designed experiments when the user does not wish to change the parameters of the model to give a full rank model. The function is not intended for situations in which the relationships between the independent variables are only approximate.

6.1. Accuracy

It should be noted that due to rounding errors a parameter that should be zero when the constraints have been imposed may be returned as a value of order *machine precision*.

6.2. References

- Golub G H and Van Loan C F (1983) *Matrix Computations* Johns Hopkins University Press, Baltimore.
- Hammarling S (1985) The Singular Value Decomposition in Multivariate Statistics *ACM Signum Newsletter* **20** (3) 2–25.
- Searle S R (1971) *Linear Models* Wiley.

7. See Also

nag_regsn_mult_linear (g02dac)
nag_regsn_mult_linear_upd_model (g02ddc)

8. Example

Data from an experiment with four treatments and three observations per treatment are read in. A model, including the mean term, is fitted by nag_regsn_mult_linear (g02dac) and the results printed. The constraint that the sum of treatment effects is zero is then read in and the parameter estimates with this constraint imposed are computed by nag_regsn_mult_linear_tran_model and printed.

8.1. Program Text

```
/* nag_regsn_mult_linear_tran_model(g02dkc) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg02.h>

#define NMAX 12
#define MMAX 5
#define TDC MMAX
#define TDQ MMAX+1
#define TDX MMAX

main()
{
    double  rss, tol;
    Integer i, iconst, ip, rank, j, m, n;
    double  df;
    Boolean svd;
    Nag_IncludeMean mean;
    char    weight, meanc;
    double  b[MMAX], c[MMAX][MMAX], cov[MMAX*(MMAX+1)/2], h[NMAX],
    p[MMAX*(MMAX+2)], q[NMAX][MMAX+1], res[NMAX], se[MMAX],
    com_ar[4*MMAX*MMAX+5*(MMAX-1)], wt[NMAX], x[NMAX][MMAX], y[NMAX];
    Integer sx[MMAX];
    double  *wtptr;

    Vprintf("g02dkc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[\n]");
    Vscanf("%ld %ld %c %c", &n, &m, &weight, &meanc);
    if (meanc=='m')
        mean = Nag_MeanInclude;
    else
        mean = Nag_MeanZero;
    if (n<=NMAX && m<MMAX)
    {
```

```

if (weight=='w')
{
    wtptr = wt;
    for (i=0; i<n; i++)
    {
        for (j=0; j<m; j++)
            Vscanf("%lf", &x[i][j]);
        Vscanf("%lf%lf", &y[i], &wt[i]);
    }
}
else
{
    wtptr = (double *)0;
    for (i=0; i<n; i++)
    {
        for (j=0; j<m; j++)
            Vscanf("%lf", &x[i][j]);
        Vscanf("%lf", &y[i]);
    }
}
for (j=0; j<m; j++)
    Vscanf("%ld", &sx[j]);
Vscanf("%ld", &ip);
/* Set tolerance */
tol = 0.00001e0;
/* Find initial estimates using g02dac */
g02dac(mean, n, (double *)x, (Integer)TDX, m, sx, ip, y, wtptr,
        &rss, &df, b, se, cov, res, h, (double *)q, (Integer)(TDQ),
        &svd, &rank, p, tol, com_ar, NAGERR_DEFAULT);

Vprintf("Estimates from g02dac\n\n");
Vprintf("Residual sum of squares = %13.4e\n", rss);
Vprintf("Degrees of freedom = %3.1f\n\n", df);
Vprintf("Variable   Parameter estimate   Standard error\n\n");
for (j=0; j<ip; j++)
    Vprintf("%6ld%20.4e%20.4e\n", j+1, b[j], se[j]);
Vprintf("\n");
/*
 *   Input constraints and call g02dkc
 */
iconst = ip - rank;
for (i=0; i<ip; ++i)
    for (j=0; j<iconst; ++j)
        Vscanf("%lf", &c[i][j]);

g02dkc(ip, iconst, p, (double *)c, (Integer)TDC, b, rss, df, se, cov,
        NAGERR_DEFAULT);

Vprintf("\n");
Vprintf("Estimates from g02dkc using constraints\n\n");
Vprintf("Variable   Parameter estimate   Standard error\n\n");
for (j=0; j<ip; j++)
    Vprintf("%6ld%20.4e%20.4e\n", j+1, b[j], se[j]);
Vprintf("\n");
}
else
{
    Vfprintf(stderr, "One or both of m and n are out of range:\n
m = %-3ld while n = %-3ld\n", m, n);
    exit(EXIT_FAILURE);
}
}
exit(EXIT_SUCCESS);
}

```

8.2. Program Data

```

g02dkc Example Program Data
 12 4 u m
 1.0 0.0 0.0 0.0 33.63
 0.0 0.0 0.0 1.0 39.62
 0.0 1.0 0.0 0.0 38.18
 0.0 0.0 1.0 0.0 41.46
 0.0 0.0 0.0 1.0 38.02
 0.0 1.0 0.0 0.0 35.83
 0.0 0.0 0.0 1.0 35.99
 1.0 0.0 0.0 0.0 36.58
 0.0 0.0 1.0 0.0 42.92
 1.0 0.0 0.0 0.0 37.80
 0.0 0.0 1.0 0.0 40.43
 0.0 1.0 0.0 0.0 37.89
 1 1 1 1 5
 0.0
 1.0
 1.0
 1.0
 1.0

```

8.3. Program Results

```

g02dkc Example Program Results
Estimates from g02dac

```

```

Residual sum of squares = 2.2227e+01
Degrees of freedom = 8.0

```

Variable	Parameter estimate	Standard error
1	3.0557e+01	3.8494e-01
2	5.4467e+00	8.3896e-01
3	6.7433e+00	8.3896e-01
4	1.1047e+01	8.3896e-01
5	7.3200e+00	8.3896e-01

```

Estimates from g02dkc using constraints

```

Variable	Parameter estimate	Standard error
1	3.8196e+01	4.8117e-01
2	-2.1925e+00	8.3342e-01
3	-8.9583e-01	8.3342e-01
4	3.4075e+00	8.3342e-01
5	-3.1917e-01	8.3342e-01
